Properties of incentre circumcentre orthocentre centroid pdf

Centers of the Triangle of Kristina Dunbar, UGA In this mission, we investigate 4 different triangle centers: centroid, circumcenter, orthocenter, and incenter. CENTROID The centroid of a triangle is constructed by taking any given triangle and connecting the mid-points of each leg of the triangle to the opposite tip. The line segment created by connecting these points is called the median. See the three medians as dotted lines in the figure below. Regardless of the shape of the triangle, the centroid will always be inside the triangle. You can look at the example above of an acute triangle or the examples below of an obtuse triangle and a straight triangle to see that this is the case. The centroid is the center of a triangle that can be considered the center of the mass. This is the balancing point to use if you want to balance a triangle on the tip of a pencil, for example. If you have Geometer's Sketchpad and want to see the GSP construction of the centroid, click here to download it. BACK to my home page. CIRCUMCENTRE The circumcentre is the center of the circle, so that all three nodes of the circle are at the same distance from the circumcentre. Thus, the circumcenter is the point that forms the origin of a circle in which all three nodes of the triangle are on the circle. Thus, the radius of the circle is the distance between the circumcenter and any of the three nodes of the triangle. It is found by finding the midpoint of each leg of the triangle and building a line perpendicular to that foot at its median point. Where all three lines intersect is the circumcenter. The circumcenter is not always inside the triangle. In fact, it can be outside the triangle, as in the case of an obtuse triangle, or it can fall in the middle of the hypotinuse of a straight triangle. See the images below for examples of this. You can see that although the circumcenter is outside the triangle in the case of the obtuse triangle, it is still equidistant from all three nodes of the triangle. If you have Geometer's Sketchpad and want to see the GSP construction of the circumcenter, click here to download it. BACK to my home page. ORTOCENTER The ortocenter is the center of the triangle created by finding the altitudes of each part. The altitude of a triangle is created by the fall of a line from each peak that is perpendicular to the opposite side. An altitude of the triangle is sometimes called height. Remember, the altitudes of a triangle do not pass through the midpoints of the legs unless you have a special triangle, like an equilateral triangle. Like the circumcenter, the orthocenter should not be inside the triangle. Check the cases of obtuse triangles and the right below. In the obtuse triangle, the orthocenter falls outside the triangle. In a straight triangle, the orthocenter falls on a top of the triangle. you have the sketchpad of the geometer and you want to see the SGP construction of the click here to download it. BACK to my home page. INCENTER Incenter is the last triangle center we investigate. It is the point that forms the origin of a circle inscribed inside the triangle. Like the centroid, the center is always inside the triangle. It is built by taking the intersection of the angular bisectors of the three nodes of the triangle. The radius of the circle is obtained by the fall of a perpendicular from the center to any of the legs of the triangle. It is illustrated below as the dotted red line. To see that the center is actually always inside the triangle, let's take a look at an obtuse triangle and a straight triangle. If you have Geometer's Sketchpad and want to see the GSP construction of the incenter, click here to download it. BACK to my home page. There is an interesting relationship between the centroid, orthocenter, and the circumcenter of a triangle. The centroid, orthocentre and circumcentre fall in a straight line. The centroid is always between the orthocenter and the circumcentre. The distance between the centroid and the orthocentre is always twice the distance between the centroid and the circumcentre. In obtuse triangles, the circumcenter is always outside the triangle opposite the highest angle. The orthocenter is always outside the triangle opposite the longest leg, on the same side with the highest angle. The only time all three of these centers fall in the same place is in the case of an equilateral triangle. In fact, in this case, the center falls in the same place, too. If you have the Geometer Sketchpad and want to see the GSP constructions of all four centers, click here to download it. This file also has all the centers together in one image, as well as the equilateral triangle. It can be used to generate all of the images above. BACK to my home page. THREE to assign 5: GSP Scripts. By Mark Ryan Each triangle has three centers - an incenter, a circumcenter, and an orthocenter - that are located at the intersection of rays, lines, and segments associated with the triangle: Incenter: If a triangle three nail bissectors intersect (an angle bissector is a radius that cuts an angle in half); the center is the center of a circle inscribed in (drawn inside) the triangle. Circumcentre: Where the three perpendicular bisectors of the sides of a triangle intersect (a perpendicular bisector is a line that forms a 90° angle with a segment and cuts the segment in half); the circumcenter is the center of a circumscribed circle about (drawn around) the triangle. Where the three altitudes of the triangle intersect. The altitude of a triangle is a segment from one tip of the triangle to the opposite side (or to the extension of the opposite side, if necessary), which is perpendicular to the opposite side; the opposite side is called the base. Finding You will find the center of a triangle at the intersection of the three angular bisectors of the triangle. This location the center of an interesting property: The center is as far away from the three sides of the triangle. No other point has this quality. Incentres, would be centroids, are always inside their triangles. The figure above shows two triangles with their incentres and inscribed circles, or circles (circles drawn inside the triangles, so that the circles barely touch the sides of each triangle). The centers are the centers of the incisors. (Don't talk about this in too much stuff if you want to be in with in-crowd.) Finding the circumcenter You will find the circumcenter of a triangle at the intersection of the perpendicular bisectors of the sides of the triangle. This location gives the circumcenter an interesting property: the circumcentre is just as far away from the three nodes of the triangle. The figure above shows two triangles with their circumcenters and circumscribed circles, or circles (circles drawn around the triangles so that the circles pass through the nodes of each triangle). Circumcenters are the centers of the circumcircles. You can see in the figure above that, unlike centroids and incentres, a circumcenter is sometimes outside the triangle. The circumcenter is inside all acute triangles Outside all the obtuse triangles on all right triangles (mid-hypopenuse) Finding the orthocenter Check out the next figure to see a pair of orthocentres. You find the orthocenter of a triangle at the intersection of its altitudes. Unlike the centroid, incenter, and circumcenter - all these are located in an interesting point of the triangle (the center of the triangle of weight, the equidistant point of the sides of the triangle, and the equidistant point of the triangle knots, respectively), the orthocenter of a triangle is not at a point with any such beautiful characteristics. Well, three out of four isn't bad. But get a load of this: Look again at the triangles in the figure. Take the four tagged points of each triangle (the three nodes plus the orthocenter). If you make a triangle out of any three of these four points, the fourth point is the orthocenter of that triangle. Pretty sweet, isn't it? Orthocentres follow the same rule as circumcentres (note that both orthocentres and circumcentres involve perpendicular lines – altitudes and perpendicular bisectors): Orthocentre is inside all acute triangles Outside of all obtuse triangles On all straight triangles (at right angle vertex) This wiki page presents some simple examples to solve triangle centers using simple properties, would be the circumcenter, the Ferat point , Brocard points, center, centroid, orthocenter, etc. One should be able to recall definitions would be circumcenter O,O,O, whose point is equidistant from all triangle nodes; incentre I,I,I, the point of which is equidistant from the sides orthocenter H,H,H, the point at which all the altitudes of the triangle intersect; centroid G,G,G, the point of intersection of the triangle medians. A One the relationship between these points is the Euler line, which states that O,G,H O, G, H O,G,H is a straight line and OG:GH=1:2 OG: GH = 1:2 OG:GH=1:2. In fact, the center of the nine point circle is also the midpoint of OH OH OH. Consider vertex AAA. Let these points coincide at the PPP. Then we know that APAPAP is the angle bisector of the BAC \angle, and is also perpendicular to BC BC. Thus, we get that ρABC=90∘−−-BAP=90∘−−-ACB =angle ABC = 90 ^ \circ - \angle BAP = 90 ^ \circ - \angle CAP = \angle ACB =90∘−-BAP=90∘−cap=-ACB. Since this is true from any peak, it means that all three angles are equal, and thus we have an equilateral triangle. To IAIAIA, IBIBB, ICICIC denotes the distance between the center and the nodes of the triangle A,B,CA,B,CA,B,C, respectively. Prove that IA-IBCA? CB+IA? ICBA-BC+IB•ICAB•AC=1\frac{IA \cdot IB}{CA \cdot CB} + \frac{IA \cdot IC}{BA \cdot BC} + \frac{IB \cdot IC}{AB \cdot AC} = 1 CA? CBIA•IB +BA? BCIA? IC +AB\ACIB? IC =1. Solution: (to be continued) Try this: Prove the Euler line, which states that O,G,H O, G, H O,G,H is cholinear. Collinear.